10.(a) In the circuit shown in Figure 6, find the currents I_1 and I_2 and the emf $\mathcal{E}_1$. State the directions of currents I_1, I_2, and I_3.

(b) Prove, numerically, that the total power produced in the circuit equals the total power consumed.

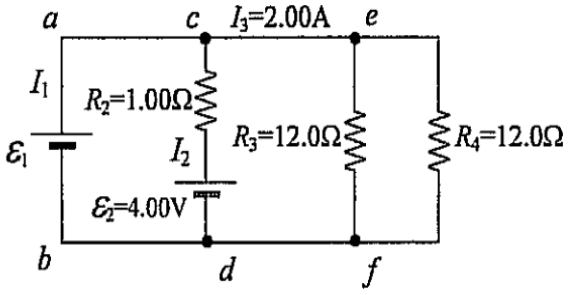

Figure 6

(a)

junctions c, d:

$$I_1 + I_2 = I_3$$
$$I_1 = I_2 - I_3$$

unknowns
$\mathcal{E}_1,\ I_2,\ I_4$

junctions e, f:

$$I_3 = I_x + I_4$$
$$I_x = I_3 - I_4$$

L: left loop

$$\mathcal{E}_1 - I_2R_2 = \mathcal{E}_2$$

M: middle loop

$$\mathcal{E}_2 - I_2R_2 - (I_3 - I_4)R_3 = 0$$

R: right loop

$$(I_3 - I_4)R_3 - I_4R_4 = 0$$

$$R: I_4 = \frac{I_3R_3}{R_3 + R_4} = \frac{(2)(12)}{12 + 12} = \boxed{1.00\,\text{A}}$$

$$M: I_2 = \frac{\mathcal{E}_2 - (I_3 - I_4)R_3}{R_2} = \frac{4 - (2 - 1)(12)}{1} = \boxed{-8.00\,\text{A}}$$

$$L: \mathcal{E}_1 = \mathcal{E}_2 - I_2R_2 = 4 - (-8) = \boxed{12.0\,\text{V}}$$

$$I_1 = I_3 - I_2 = 2 - (-8) = \boxed{10.0\,\text{A}}$$

$$I_x = I_3 - I_4 = 2 - 1 = \boxed{1.00\,\text{A}}$$

(b)

$$P_{\text{in}} = I_1\mathcal{E}_1 + I_2\mathcal{E}_2 = (10\,\text{A})(12\,\text{V}) + (-8\,\text{A})(4\,\text{V}) = \boxed{88\,\text{W}}$$

$$P_{\text{out}} = I_2^2R_2 + I_x^2R_3 + I_4^2R_4 = (8\,\text{A})^2(1\,\Omega) + (1\,\text{A})^2(12\,\Omega) + (1\,\text{A})^2(12\,\Omega) = \boxed{88\,\text{W}}$$